

Constraining the High-Density Behavior of Nuclear Symmetry Energy with the Tidal Polarizability of Neutron Stars

F. J. Fattoyev,^{1,2,*} J. Carvajal,^{1,3,†} W. G. Newton,^{1,‡} and Bao-An Li^{1,4,§}

¹*Department of Physics and Astronomy, Texas A&M University-Commerce, Commerce, Texas 75429-3011, USA*

²*Institute of Nuclear Physics, Tashkent 100214, Uzbekistan*

³*Miami Dade College, 10525 SW 42nd TERR, Miami, Florida 33165, USA*

⁴*Department of Applied Physics, Xian Jiao Tong University, Xian 710049, China*

(Dated: October 15, 2012)

Using a set of model equations of state satisfying the latest constraints from both terrestrial nuclear experiments and astrophysical observations as well as state-of-the-art nuclear many-body calculations of the pure neutron matter equation of state, the tidal polarizability of canonical neutron stars in coalescing binaries is found to be a very sensitive probe of the high-density behavior of nuclear symmetry energy which is among the most uncertain properties of dense neutron-rich nucleonic matter. Moreover, it changes less than $\pm 10\%$ by varying various properties of symmetric nuclear matter and symmetry energy around the saturation density within their respective ranges of remaining uncertainty.

PACS numbers: 26.60.Kp, 21.65.Mn, 26.60.-c, 04.40.Dg, 97.60.Jd, 97.80.-d, 95.85.Sz

Introduction.—Understanding the nature of neutron-rich nucleonic matter is a major thrust of current research in both nuclear physics and astrophysics [1]. To realize this goal, many experiments and observations are being carried out or proposed using a wide variety of advanced new facilities, such as, Facilities for Rare Isotope Beams (FRIB), X-ray satellites and gravitational wave detectors. Most critical to interpreting results of these experiments and observations is the equation of state (EOS) of neutron-rich nucleonic matter, i.e., $E(\rho, \alpha) = E_0(\rho) + S(\rho)\alpha^2 + \mathcal{O}(\alpha^4)$, where $E(\rho, \alpha)$ and $E_0(\rho)$ are the specific energy in asymmetric nuclear matter of isospin asymmetry $\alpha = (\rho_n - \rho_p)/\rho$ and symmetric nuclear matter (SNM), respectively, and $S(\rho)$ is the symmetry energy encoding the energy cost of converting all protons in SNM to neutrons. Thanks to the continuing efforts of both the nuclear physics and astrophysics community over several decades, the EOS of SNM around the saturation density ρ_0 has been well constrained. Moreover, combining information from studying the collective flow and kaon production in relativistic heavy-ion collisions in several terrestrial nuclear physics laboratories [2] and the very recent discovery of the maximum mass of neutron stars [3], the EOS of SNM has been limited in a relatively small range up to about $4.5\rho_0$. The symmetry energy $S(\rho)$ is a vital ingredient in describing the structure of rare isotopes and their reaction mechanisms. It also determines uniquely the proton fraction and thus the cooling mechanism, appearance of hyperons and possible kaon condensation in neutron stars. Moreover, it affects significantly the structure, such as the radii, moment of inertia and the core-crust transition density, as well as the frequencies and damping times of various oscillation modes of neutron stars, see, e.g., Refs. [4, 5] for reviews. Intensive efforts devoted to constraining $S(\rho)$ using various approaches have recently led

to a close merger around $S(\rho_0) \approx 30$ MeV and its density slope $L \equiv 3\rho_0 (dS(\rho)/d\rho)_{\rho_0} \approx 50$ MeV with a few exceptions, although the error bars for L from different approaches may vary broadly [6–12]. On the other hand, the high-density behavior of $S(\rho)$ remains very uncertain despite its importance to understanding what happens in the core of neutron stars [13–17] and in reactions with high energy radioactive beams [18]. The predictions for the high-density behavior of the symmetry energy from all varieties of nuclear models diverge dramatically [19–21], with some models predicting very stiff symmetry energies that increase continuously with density [21–26], and others predicting relatively soft ones, or an $S(\rho)$ that first increases with density, then saturates and starts decreasing with increasing density [19, 27–41]. These uncertainties can be traced to our poor knowledge about the isospin dependence of strong interaction in dense neutron-rich medium, particularly the spin-isospin dependence of three-body and many-body forces, the short-range behavior of nuclear tensor force and the isospin dependence of nucleon-nucleon correlations in dense medium, see, e.g. Refs. [42, 43]. Little experimental progress has been made in constraining the high density $S(\rho)$ partially because of the lack of sensitive probes. While several observables have been proposed [18] and some indications of the high-density $S(\rho)$ have been reported recently [44, 45], conclusions based on terrestrial nuclear experiments remain controversial [46]. To our best knowledge, the only astrophysical probe for high density $S(\rho)$ proposed so far is the late time neutrino signal from a core collapse supernova [47]. In this Letter, we show that the tidal polarizability of canonical neutron stars in the coalescing binaries is a very sensitive probe of the high-density behavior of nuclear symmetry energy independent of the remaining uncertainties of the SNM EOS and $S(\rho)$ near saturation density.

Stellar structure and tidal polarizability. — Coalescing binary neutron stars are among the most promising sources of gravitational waves (GW). One of the most important features of binary mergers is the tidal deformation neutron stars undergo as they approach each other prior to merger, the strength of which can give us precious information about the neutron-star matter EOS [48–59]. At the early stage of an inspiral tidal effects may be effectively described through the tidal polarizability parameter λ [48, 51–53] defined via $Q_{ij} = -\lambda \mathcal{E}_{ij}$, where Q_{ij} is the induced quadrupole moment of a star in binary, and \mathcal{E}_{ij} is the static external tidal field of the companion star. The tidal polarizability can be expressed in terms of the dimensionless tidal Love number k_2 and the neutron star radius R as $\lambda = 2k_2 R^5 / (3G)$. The tidal Love number k_2 is found using the following expression [49, 54]:

$$k_2 = \frac{1}{20} \left(\frac{R_s}{R} \right)^5 \left(1 - \frac{R_s}{R} \right)^2 \left[2 - y_R + (y_R - 1) \frac{R_s}{R} \right] \times \\ \times \left\{ \frac{R_s}{R} \left(6 - 3y_R + \frac{3R_s}{2R} (5y_R - 8) + \frac{1}{4} \left(\frac{R_s}{R} \right)^2 \left[26 - \right. \right. \right. \\ \left. \left. \left. - 22y_R + \left(\frac{R_s}{R} \right) (3y_R - 2) + \left(\frac{R_s}{R} \right)^2 (1 + y_R) \right] \right) + \right. \\ \left. + 3 \left(1 - \frac{R_s}{R} \right)^2 \left[2 - y_R + (y_R - 1) \frac{R_s}{R} \right] \times \right. \\ \left. \times \log \left(1 - \frac{R_s}{R} \right) \right\}^{-1}, \quad (1)$$

where $R_s \equiv 2M$ is the Schwarzschild radius of the star, and $y_R \equiv y(R)$ can be calculated by solving the following first-order differential equation:

$$r \frac{dy(r)}{dr} + y(r)^2 + y(r)F(r) + r^2 Q(r) = 0, \quad (2)$$

with

$$F(r) = \frac{r - 4\pi r^3 (\mathcal{E}(r) - P(r))}{r - 2M(r)}, \quad (3)$$

$$Q(r) = \frac{4\pi r \left(5\mathcal{E}(r) + 9P(r) + \frac{\mathcal{E}(r) + P(r)}{\partial P(r)/\partial \mathcal{E}(r)} - \frac{6}{4\pi r^2} \right)}{r - 2M(r)} - \\ - 4 \left[\frac{M(r) + 4\pi r^3 P(r)}{r^2 (1 - 2M(r)/r)} \right]^2. \quad (4)$$

The Eq. (2) must be integrated together with the Tolman-Oppenheimer-Volkoff (TOV) equation. Given the boundary conditions in terms of $y(0) = 2$, $P(0) = P_c$ and $M(0) = 0$, the tidal Love number can be obtained once an EOS is supplied. Previous studies have used both polytropic EOSs and several popular nuclear EOSs available in the literature [48–59]. While other particles may be present, for the purpose of this work, it is sufficient to assume that neutron stars consist of only

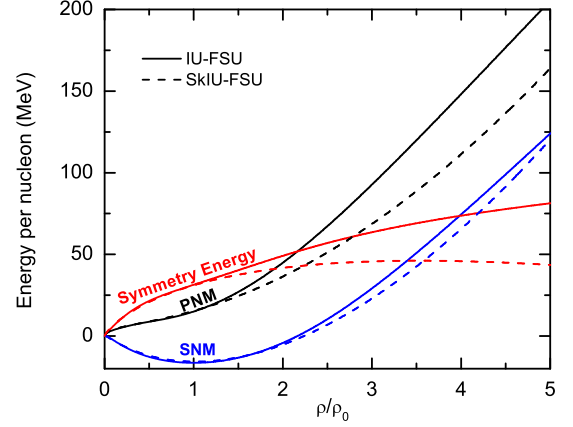


FIG. 1: (Color online) The EOS of SNM and PNM as well as the symmetry energy as a function of density obtained within the IU-FSU RMF model and the SHF approach using the SkIU-FSU parameter set.

neutrons (n), protons (p), electrons (e) and muons (μ) in β -equilibrium.

Constrained EOS of neutron-rich nuclear matter. — We use two classes of nuclear EOSs within the Relativistic Mean Field (RMF) model and the Skyrme Hartree-Fock (SHF) approach. All of these EOSs are adjusted to satisfy the following four conditions within their respective uncertain ranges: (1) reproducing the PNM EOS at sub-saturation densities predicted by the latest state-of-the-art microscopic nuclear many-body theories [28, 60–65]; (2) predicting correctly saturation properties of symmetric nuclear matter, i.e. nucleon binding energy $B = -16 \pm 1$ MeV and incompressibility $K_0 = 230 \pm 20$ MeV and nucleon effective mass $M_0^* = 0.61 \pm 0.03 M$ at saturation density $\rho_0 = 0.155 \pm 0.01 \text{ fm}^{-3}$; (3) predicting a fiducial value of symmetry energy $S(2\rho_0/3) = 26 \pm 0.5$ MeV, $J \equiv S(\rho_0) = 31 \pm 2$ MeV and the density slope of symmetry energy $L = 50 \pm 10$ MeV; (4) passing through the terrestrial constraints on the EOS of SNM between $2\rho_0$ and $4.5\rho_0$ [2] and giving a maximum mass of neutron stars of about $2M_\odot$ assuming they are made of only the $npe\mu$ matter without considering other degrees of freedom or invoking any exotic mechanism [3, 66]. As an example, two such EOSs obtained using the IU-FSU RMF model [67] and the SHF using the SkIU-FSU parameter set [12] are shown in Fig. 1. By design, they both have the same EOS for SNM and PNM around and below ρ_0 . Thus, at sub-saturation densities the values of $S(\rho)$ which is approximately the difference between the EOSs for PNM and SNM are almost identical for the two models. However, the values of $S(\rho)$ are significantly different above about $1.5\rho_0$ with the IU-FSU leading to a much stiffer $S(\rho)$ at high densities. More quantitatively, the $S(\rho)$ with IU-FSU is 40 – 60% higher in the density

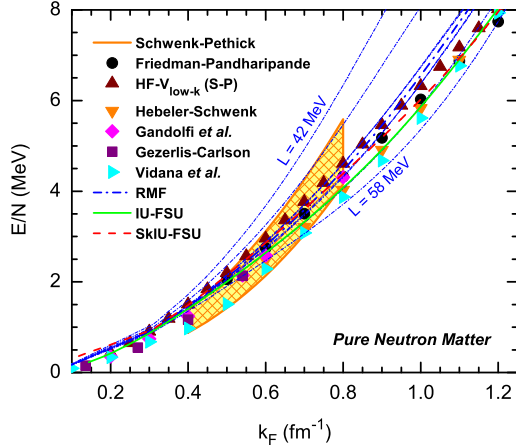


FIG. 2: (Color online) Energy per nucleon as a function of the Fermi momentum for PNM for selected models described in the text.

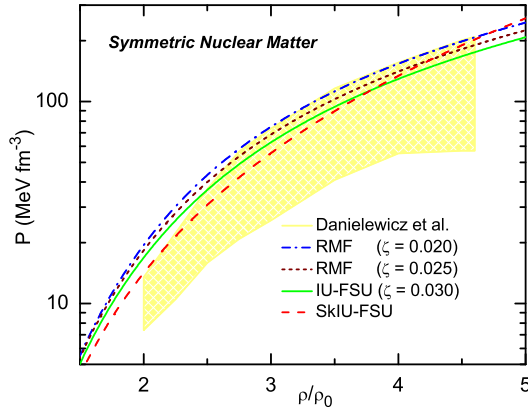


FIG. 3: (Color online) The pressure of SNM given as the function of baryon density. Here ρ_0 is the nuclear matter saturation density and the shaded area represents the EOS extracted from the analysis of [2].

range of $\rho/\rho_0 = 3 - 4$ expected to reach in the core of canonical neutron stars.

To test the sensitivity of the tidal polarizability to variations of properties of neutron-rich nuclear matter around ρ_0 within the constraints listed above, we build 17 RMF parameterizations by systematically varying the values of K_0 , M_0^* , L , and the ζ parameter of the RMF model that controls the omega-meson self interactions [68] and subsequently the high-density component of the EOS of SNM. Besides the constraints listed above, all parameter sets can correctly reproduce the experimental values for the binding energy and charge radius of ^{208}Pb and the ground state properties of other closed

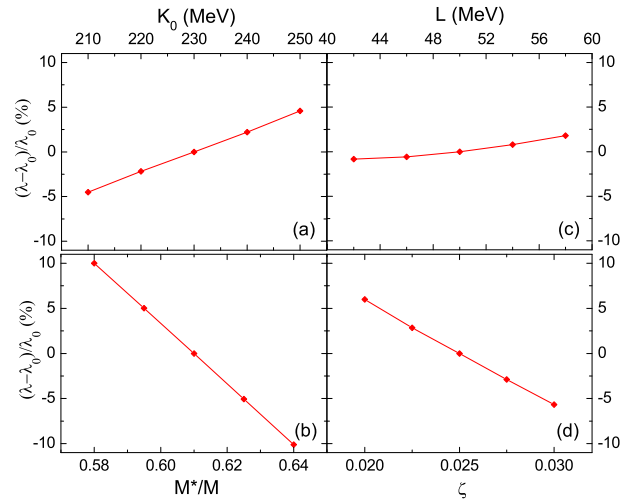


FIG. 4: (Color online) Percentage changes in the tidal polarizability of a 1.4 solar mass neutron star by individually varying properties of nuclear matter K_0 (a), M^* (b), L (c), and the ζ parameter (d) of the RMF model with respect to the value using the base model.

shell nuclei within 2% uncertainty [69]. As a reference for comparisons, we select $K_0 = 230$ MeV, $M_0^* = 0.61 M$, $L = 50$ MeV, and $\zeta = 0.025$ for our base model, which predicts $\rho_0 = 0.1524 \text{ fm}^{-3}$, $B = -16.33$ MeV and $J = 31.64$ MeV. The representative model EOSs for PNM at sub-saturation densities and those for SNM at supra-saturation densities are compared with their constraints in Fig. 2 and Fig. 3, respectively. It is seen that the SkIU-FSU and all the RMF models with $42 < L < 58$ MeV can satisfy the PNM EOS constraint. Also, they can all satisfy simultaneously the high density SNM EOS constraint with $0.02 < \zeta < 0.03$. Moreover, they all give a maximum mass for neutron stars between $1.94M_\odot$ and $2.07M_\odot$ and radii between 12.33 km and 13.22 km for canonical neutron stars [67] consistent with existing observations [3, 66, 70].

Results and discussions. —First, we examine sensitivities of the tidal polarizability λ of a $1.4M_\odot$ neutron star to the variations of SNM properties and the slope of the symmetry energy around ρ_0 in Fig. 4. The changes of λ relative to the values for our base RMF model are shown for the remaining RMF EOSs. It is very interesting to see that the tidal polarizability is rather insensitive to the variation of L although it changes up to $\pm 10\%$ with K_0 , M^* and ζ within their individual uncertain ranges. While the averaged mass is $M = 1.33 \pm 0.05 M_\odot$, neutron stars in binaries have a broad mass distribution [71]. It is thus necessary to investigate the mass dependence of the tidal polarizability. Whereas what can be measured for a neutron star binary of mass M_1 and M_2 is the mass-weighted polarizability $\tilde{\lambda} = [\lambda_1 (M_1 + 12M_2) / M_1 + 1 \leftrightarrow 2] / 26$ [53], for the purpose of this study it is sufficient to consider bi-

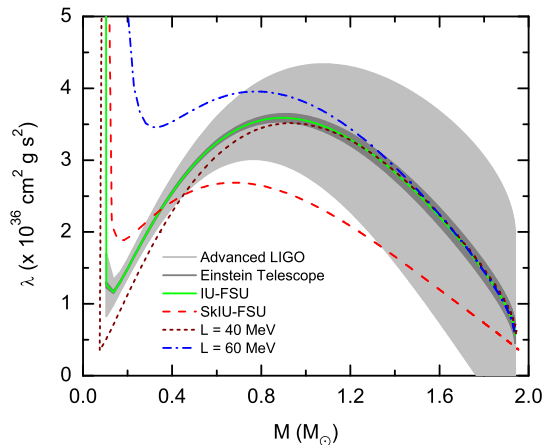


FIG. 5: (Color online) Tidal polarizability λ of a single neutron star as a function of neutron-star mass for a range of EOS that allow various stiffness of symmetry energies. A crude estimate of uncertainties in measuring λ for equal mass binaries at a distance of $D = 100$ Mpc is shown for the Advanced LIGO (shaded light-grey area) and the Einstein Telescope (shaded dark-grey area).

naries consisting of two neutron stars with equal masses. What can we learn from the tidal polarizability of light and massive neutron stars, respectively? Shown in Fig. 5 are the tidal polarizability λ as a function of neutron-star mass for a range of EOSs. Most interestingly, it is seen that the IU-FSU and SkIU-FSU models which are different only in their predictions for the nuclear symmetry energy above about $1.5\rho_0$ as shown in Fig. 1 lead to significantly different λ values in a broad mass range from 0.5 to $2 M_\odot$. More quantitatively, a 41% change in λ from 2.828×10^{36} (IU-FSU) to 1.657×10^{36} (SkIU-FSU) is observed for a canonical neutron star of $1.4 M_\odot$. For a comparison, we notice that this effect is as strong as the symmetry energy effect on the late time neutrino flux from the cooling of proto-neutron stars [47]. Moreover, it is shown that the variation of L has a very small effect on the tidal polarizability λ of massive neutron stars, which is consistent with the results shown in Fig. 4. On the other hand, the L parameter affects significantly the tidal polarizability of neutron stars with $M \leq 1.2 M_\odot$. These observations can be easily understood. From Eq. (1) the Love number k_2 is essentially determined by the compactness parameter M/R and the function $y(R)$. Both of them are obtained by integrating the EOS all the way from the core to the surface. Since the saturation density approximately corresponds to the central density of a $0.3 M_\odot$ neutron star, one thus should expect that only the Love number of low-mass neutron stars to be sensitive to the EOS around the saturation density. However, for canonical and more massive neutron stars, the central density is higher than $3 - 4\rho_0$, and therefore both

the compactness M/R and $y(R)$ show stronger sensitivity to the variation of EOS at supra-saturation densities. Since all the EOSs for SNM at supra-saturation densities have already been constrained by the terrestrial nuclear physics data and required to give a maximum mass about $2 M_\odot$ for neutron stars, the strongest effect on calculations of the tidal polarizability of massive neutron stars should therefore come from the high-density behavior of the symmetry energy.

It has been suggested that the Advanced LIGO-Virgo detector may potentially measure the tidal polarizability of binary neutron stars with a moderate accuracy. Are the existing or planned GW detectors sensitive enough to measure the predicted effects of high-density symmetry energy on the tidal polarizability? To answer this question, as an example we estimate uncertainties in measuring λ for equal mass binaries at an optimally-oriented distance of $D = 100$ Mpc [53, 72] using the same approach as detailed in Refs. [53, 59]. These are shown for the Advanced LIGO-Virgo (shaded light-grey area) and the Einstein Telescope (shaded dark-grey area) in Fig. 5. It is seen that the Advanced LIGO-Virgo will unlikely constrain the EOS and symmetry energy at supra-saturation densities within the estimated uncertainty, although it is possible that a rare but nearby binary system may be found and provide a much more tighter constraint [53]. Nevertheless, measurements for binaries consisting of light neutron stars can still help further constrain the symmetry energy around the saturation density. On the other hand, it is exciting to see that the narrow uncertain range for the proposed Einstein Telescope will enable it to tightly constrain the symmetry energy especially at high densities.

Conclusions.—Using the EOSs for neutron-rich nucleonic matter satisfying the latest constraints from both terrestrial nuclear experiments and astrophysical observations, as well as the state-of-the-art nuclear many-body calculations for PNM EOS, we found that the tidal polarizability of canonical neutron stars in coalescing binaries is very sensitive to the high-density behavior of nuclear symmetry energy, but little affected by the variations of SNM EOS and symmetry energy around the saturation density within their remaining uncertainty ranges. Future measurements of the tidal polarizability of neutron stars using the proposed Einstein Telescope will help constrain stringently the high-density behavior of nuclear symmetry energy, and thus the nature of dense neutron-rich nucleonic matter.

Acknowledgments. —We would like to thank Dr. J. Piekarewicz and Dr. Jun Xu for various useful discussions. This work is supported in part by the National Aeronautics and Space Administration under grant NNX11AC41G issued through the Science Mission Directorate, and the National Science Foundation under Grants No. PHY-0757839, No. PHY-1062613 and No. PHY-1068022.

-
- * Electronic address: Farrooh.Fattoyev@tamuc.edu
† Electronic address: Jose.Carvajal002@mymdc.net
‡ Electronic address: William.Newton@tamuc.edu
§ Electronic address: Bao-An.Li@tamuc.edu
- [1] The 2007 US NSAC Long Range Plan, URL <http://www.er.doe.gov/np/nsac/nsac.html>.
 - [2] P. Danielewicz, R. Lacey, and W. G. Lynch, *Science* **298**, 1592 (2002).
 - [3] P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts, and J. W. T. Hessels, *Nature* **467**, 1081 (2010).
 - [4] J. M. Lattimer and M. Prakash, *Phys. Rept.* **333**, 121 (2000).
 - [5] J. M. Lattimer and M. Prakash, *Science* **304**, 536 (2004).
 - [6] C. Xu, B.-A. Li, and L.-W. Chen, *Phys. Rev.* **C82**, 054607 (2010).
 - [7] W. G. Newton, M. Gearheart, and B.-A. Li (2011), 1110.4043.
 - [8] A. W. Steiner and S. Gandolfi, *Phys. Rev. Lett.* **108**, 081102 (2012).
 - [9] J. M. Lattimer and Y. Lim (2012), 1203.4286.
 - [10] M. Dutra, O. Lourenço, J. S. Sà Martins, A. Delfino, J. R. Stone, and P. D. Stevenson, *Phys. Rev.* **C85**, 035201 (2012).
 - [11] M. Tsang *et al.*, *Phys. Rev.* **C86**, 015803 (2012).
 - [12] F. J. Fattoyev, W. G. Newton, J. Xu, and B.-A. Li, *Phys. Rev.* **C86**, 025804 (2012).
 - [13] M. Kutschera, *Phys. Lett.* **B340**, 1 (1994).
 - [14] S. Kubis and M. Kutschera, *Acta Phys. Polon.* **B30**, 2747 (1999).
 - [15] S. Kubis and M. Kutschera, *Nucl. Phys.* **A720**, 189 (2003).
 - [16] D.-H. Wen, B.-A. Li, and L.-W. Chen, *Phys. Rev. Lett.* **103**, 211102 (2009).
 - [17] H. K. Lee, B.-Y. Park, and M. Rho, *Phys. Rev.* **C83**, 025206 (2011).
 - [18] B.-A. Li, L.-W. Chen, and C. M. Ko, *Phys. Rept.* **464**, 113 (2008).
 - [19] B. A. Brown, *Phys. Rev. Lett.* **85**, 5296 (2000).
 - [20] A. Szmaglinski, W. Wojcik, and M. Kutschera, *Acta Phys. Polon.* **B37**, 277 (2006).
 - [21] A. E. L. Dieperink, Y. Dewulf, D. Van Neck, M. Waroquier, and V. Rodin, *Phys. Rev.* **C68**, 064307 (2003).
 - [22] A. W. Steiner, M. Prakash, J. M. Lattimer, and P. J. Ellis, *Phys. Rept.* **411**, 325 (2005).
 - [23] C.-H. Lee, T. T. S. Kuo, G. Q. Li, and G. E. Brown, *Phys. Rev.* **C57**, 3488 (1998).
 - [24] C. J. Horowitz and J. Piekarewicz, *Phys. Rev. Lett.* **86**, 5647 (2001).
 - [25] L.-W. Chen, C. M. Ko, and B.-A. Li, *Phys. Rev.* **C76**, 054316 (2007).
 - [26] Z. H. Li, U. Lombardo, H.-J. Schulze, W. Zuo, L. W. Chen, and H. R. Ma, *Phys. Rev.* **C74**, 047304 (2006).
 - [27] V. R. Pandharipande and V. K. Garde, *Phys. Lett.* **B39**, 608 (1972).
 - [28] B. Friedman and V. R. Pandharipande, *Nucl. Phys.* **A361**, 502 (1981).
 - [29] R. B. Wiringa, V. Fiks, and A. Fabrocini, *Phys. Rev.* **C38**, 1010 (1988).
 - [30] N. Kaiser, S. Fritsch, and W. Weise, *Nucl. Phys.* **A697**, 255 (2002).
 - [31] P. G. Krastev and F. Sammarruca, *Phys. Rev.* **C74**, 025808 (2006).
 - [32] E. Chabanat, J. Meyer, P. Bonche, R. Schaeffer, and P. Haensel, *Nucl. Phys.* **A627**, 710 (1997).
 - [33] J. R. Stone, J. C. Miller, R. Konciewicz, P. D. Stevenson, and M. R. Strayer, *Phys. Rev.* **C68**, 034324 (2003).
 - [34] L.-W. Chen, C. M. Ko, and B.-A. Li, *Phys. Rev.* **C72**, 064309 (2005).
 - [35] J. Decharge and D. Gogny, *Phys. Rev.* **C21**, 1568 (1980).
 - [36] C. B. Das, S. D. Gupta, C. Gale, and B.-A. Li, *Phys. Rev.* **C67**, 034611 (2003).
 - [37] D. T. Khoa, W. von Oertzen, and A. A. Ogloblin, *Nucl. Phys.* **A602**, 98 (1996).
 - [38] D. N. Basu, P. R. Chowdhury, and C. Samanta, *Nucl. Phys.* **A811**, 140 (2008).
 - [39] W. D. Myers and W. J. Swiatecki, *Acta Phys. Polon.* **B26**, 111 (1995).
 - [40] S. Banik and D. Bandyopadhyay, *J. Phys.* **G26**, 1495 (2000).
 - [41] P. R. Chowdhury, D. N. Basu, and C. Samanta, *Phys. Rev.* **C80**, 011305 (2009).
 - [42] C. Xu and B.-A. Li, *Phys. Rev.* **C81**, 064612 (2010).
 - [43] C. Xu, A. Li, and B.-A. Li (2012), 1207.1639.
 - [44] Z. Xiao, B.-A. Li, L.-W. Chen, G.-C. Yong, and M. Zhang, *Phys. Rev. Lett.* **102**, 062502 (2009).
 - [45] P. Russotto *et al.*, *Phys. Lett.* **B697**, 471 (2011).
 - [46] W. Trautmann and H. H. Wolter, *Int. J. Mod. Phys.* **E21**, 1230003 (2012).
 - [47] L. F. Roberts, G. Shen, V. Cirigliano, J. A. Pons, S. Reddy, and S. E. Woosley, *Phys. Rev. Lett.* **108**, 061103 (2012).
 - [48] É. É. Flanagan and T. Hinderer, *Phys. Rev.* **D77**, 021502 (2008).
 - [49] T. Hinderer, *Astrophys. J.* **677**, 1216 (2008).
 - [50] T. Binnington and E. Poisson, *Phys. Rev.* **D80**, 084018 (2009).
 - [51] T. Damour and A. Nagar, *Phys. Rev.* **D80**, 084035 (2009).
 - [52] T. Damour and A. Nagar, *Phys. Rev.* **D81**, 084016 (2010).
 - [53] T. Hinderer, B. D. Lackey, R. N. Lang, and J. S. Read, *Phys. Rev.* **D81**, 123016 (2010).
 - [54] S. Postnikov, M. Prakash, and J. M. Lattimer, *Phys. Rev.* **D82**, 024016 (2010).
 - [55] L. Baiotti, T. Damour, B. Giacomazzo, A. Nagar, and L. Rezzolla, *Phys. Rev. Lett.* **105**, 261101 (2010).
 - [56] L. Baiotti, T. Damour, B. Giacomazzo, A. Nagar, and L. Rezzolla, *Phys. Rev.* **D84**, 024017 (2011).
 - [57] B. D. Lackey, K. Kyutoku, M. Shibata, P. R. Brady, and J. L. Friedman, *Phys. Rev.* **D85**, 044061 (2012).
 - [58] F. Pannarale, L. Rezzolla, F. Ohme, and J. S. Read, *Phys. Rev.* **D84**, 104017 (2011).
 - [59] T. Damour, A. Nagar, and L. Villain, *Phys. Rev.* **D85**, 123007 (2012).
 - [60] A. Schwenk and C. J. Pethick, *Phys. Rev. Lett.* **95**, 160401 (2005).
 - [61] E. N. E. van Dalen and H. Muther, *Phys. Rev.* **C80**, 037303 (2009).
 - [62] K. Hebeler and A. Schwenk, *Phys. Rev.* **C82**, 014314 (2010).
 - [63] S. Gandolfi, A. Y. Illarionov, S. Fantoni, F. Pederiva, and K. E. Schmidt, *Phys. Rev. Lett.* **101**, 132501 (2008).
 - [64] A. Gezerlis and J. Carlson, *Phys. Rev.* **C81**, 025803 (2010).

- [65] I. Vidana, A. Polls, and A. Ramos, Phys. Rev. **C65**, 035804 (2002).
- [66] A. W. Steiner, J. M. Lattimer, and E. F. Brown, Astrophys. J. **722**, 33 (2010).
- [67] F. J. Fattoyev, C. J. Horowitz, J. Piekarewicz, and G. Shen, Phys. Rev. **C82**, 055803 (2010).
- [68] H. Mueller and B. D. Serot, Nucl. Phys. **A606**, 508 (1996).
- [69] B. G. Todd and J. Piekarewicz, Phys. Rev. **C67**, 044317 (2003).
- [70] F. Özel, G. Baym, and T. Güver, Phys. Rev. **D82**, 101301 (2010).
- [71] F. Özel, D. Psaltis, R. Narayan, and A. S. Villarreal, Astrophys. J. **757**, 55 (2012).
- [72] J. Abadie *et al.*, (LIGO Scientific Collaboration, Virgo Collaboration), Class. Quant. Grav. **27**, 173001 (2010).